This mechanism for failure is not present in James' method. No matter how bad an intermediate result in the iterations may be, the points cannot get out of order. In James' method, the arc lengths of points in the z plane corresponding to equally spaced points in the ζ plane are determined by integrating the calculated values of the modulus of the mapping derivative. Since the integrand is always a positive number, the ordering of the points cannot be changed. As a result, James' method can be used successfully for considerably more complicated cases than Theodorsen's method can.

Acknowledgment

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Flow in Streamwise Corners Having Large Transverse Curvature

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Introduction

A SEMI-INFINITE surface that is everywhere plane except for a single line on which the curvature is infinite is conveniently called a sharp corner. When the line is straight and parallel with an infinite stream in which it is immersed it is called a sharp streamwise corner and is a configuration considered by many workers, e.g., Refs. 1-4. The flow in streamwise corners is interesting in the three-dimensional effects present near to the surface discontinuity, and in its relevance to systems of engineering importance. One particularly interesting feature of the sharp corner is the appearance of the streamwise velocity component as seen in the symmetry plane of the corner. Even in the absence of a freestream pressure gradient this is reminiscent of the profile at the point of separation in two-dimensional flow and consequently suggests a precariously stable condition in the corner flow. The separation-like appearance is directly related to the infinite curvature and the no-slip condition governing the flow at the corner surface. On the other hand, the three-dimensionality in the flow is not conditional on the curvature being infinite but only requires that the curvature should be at least of the same order as the reciprocal of the adjacent two-dimensional boundary-layer thickness δ. It may, therefore, be worthwhile to relax the infinity condition and require only that it be at least $0(\delta^{-1})$; the three-dimensionality will be retained, the unstable appearance of the streamwise velocity profile may be helpfully modified, and the problem to be solved may even be a little more realistic in that practical corners frequently incorporate a small root radius and are seldom absolutely sharp.

These considerations prompted the work presented here, but instead of attempting the "radiused" corner in all its formidable generality, a special case is considered that permits the use of a flow similarity hypothesis and the simplifications arising therefrom. Specifically, the transverse radius of curvature is assumed to be independent of the streamwise coordinates in a suitably transformed coordinate system.

Outline Solution

Consider two imaginary quarter-infinite planes joined at an edge parallel to an infinite uniform stream of speed U and coincident with the positive axis y^1 of a right-handed Cartesian (y^1,y^2,y^3) coordinate system. Let y^1 point downstream and let the planes be symmetrically disposed with respect to the y^2 axis. The analysis will be concerned with the flow along a semi-infinite material surface that everywhere lies in the quarter-infinite planes except for a small region near their joining line in which the surface has a region of finite curvature effecting a smooth transition between its two asymptotes. The arrangement is shown in Fig. 1.

Also shown in Fig. 1 is a curvilinear coordinate system (x^1,x^2,x^3) in which the radiused corner is the surface $x^2=0$. The x^i system is now transformed to a ξ^i system by the relations $\xi^1=x^1$, $\xi^j=(U/\nu x^1)^{1/2}x^j$ $(j=2,3)$; and the y^i to ξ^i transformation is

$$y^1=\xi^1=x^1\equiv x$$

$$y^2=(\nu x/U)^{1/2}\left\{\int_0^\infty(\tan\lambda^*\cos\lambda-\sin\lambda)\,d\xi^3+\xi^2+\int_0^{\xi^3}\sin\lambda d\xi^3\right\}\equiv(\nu x/U)^{1/2}I_1$$

$$y^3=(\nu x/U)^{1/2}\int_0^{\xi^3}\cos\lambda d\xi^3\equiv(\nu x/U)^{1/2}I_2 \qquad (1)$$

ν is the kinematic viscosity; λ is the angle between the tangent to the surface and the y^3 axis; and it is assumed that $\lambda(x^1,x^3)=\lambda(\xi^3)$, $\lambda(\xi^3)=-\lambda(-\xi^3)$, and that $\lambda\rightarrow\lambda^*$ exponentially fast with increasing ξ^3.

For the given geometry it is expected that a similar solution for the velocity distribution will exist in which the leading terms in expansions of the physical velocity components in the ξ^i direction will be of the form $R_e^{-n_i}v(i)(\xi^2,\xi^3)U$, with

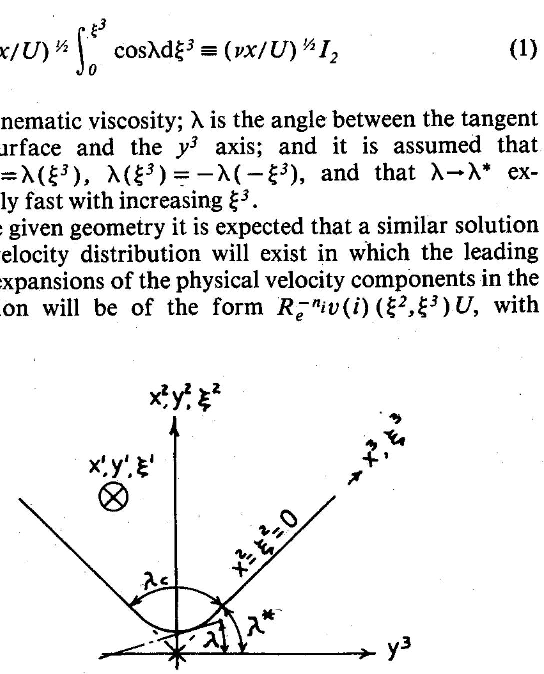

Fig. 1 Curvilinear (x^1,x^2,x^3) and Cartesian (y^1,y^2,y^3) coordinate systems.

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$R_e \equiv Ux/\nu$ and $n_1 = 0$, $n_2 = n_3 = 1/2$. The assumption that this is true is implicit in the derivation of the boundary-layer equations from the tensor form of the Navier-Stokes equations, and in this exercise the procedure described by Desai and Mangler[2] has been followed. It has been verified that Desai and Mangler's equations (26-29) are applicable here, and, when expressed in terms of the present problem, they become

$$\Delta u - (\gamma_{23},_3 + \psi)u,_2 + (\gamma_{22},_3 - \phi)u,_3 = 0 \qquad (2)$$

$$\Delta A - (\gamma_{23},_3 + \psi)A,_2 + (\gamma_{22},_3 - \phi)A,_3 + (uJ)A$$
$$= j_{11},_3 uu,_2 - j_{11},_2 uu,_3 \qquad (3)$$

$$\psi,_2 + \phi,_3 = -Ju \qquad (4)$$

$$JA = (\gamma_{23}\phi + \gamma_{22}\psi),_3 - (\gamma_{33}\phi + \gamma_{23}\psi),_2 \qquad (5)$$

In these equations

$$\Delta \equiv \gamma_{33}\partial^2/(\partial\xi^2)^2 - 2\gamma_{23}\partial^2/(\partial\xi^2\partial\xi^3) + \gamma_{22}\partial^2/(\partial\xi^3)^2$$

$$\gamma_{22} = \gamma_{33} = J^{-1} = \sec\lambda, \quad \gamma_{23} = \tan\lambda$$

$$,_i \equiv \partial/\partial\xi^i \qquad (i = 2,3)$$

and $u(\xi^2,\xi^3)$, $\psi(\xi^2,\xi^3)$, $\phi(\xi^2,\xi^3)$ are by definition related to the contravariant velocity components v^i by

$$v^1 = uU, \qquad v^2 = \frac{\psi}{J}\frac{U}{x}, \qquad v^3 = \frac{\phi}{J}\frac{U}{x}$$

Equation (5) is the defining equation for $A(\xi^2,\xi^3)$, which is related to the streamwise component of vorticity ω by

$$\omega = (U/x)[1/2(I_1\tan\lambda + I_2)u,_2 - 1/2 I_1 \sec\lambda u,_3 - A]$$

Greater convenience in the numerical method of solution to be employed is obtained if Eqs. (4) and (5) are replaced by an equivalent pair of second-order equations contrived in the following way: Differentiate Eq. (4) with respect to ξ^3 and multiply the result by γ_{22}, giving an equation (a) (not reproduced here). Differentiate Eq. (5) with respect to ξ^2 and subtract the result from Eq. (a) to get

$$\Delta\phi + \gamma_{23}\phi,_{23} - \gamma_{23},_3\phi,_2 + \gamma_{23}\psi,_{22} - \gamma_{22},_3\psi,_2$$
$$= -(JA),_2 - \gamma_{22}(Ju),_3 \qquad (6)$$

Similarly, differentiate Eq. (4) with respect to ξ^2, multiply by γ_{33}, and subtract the result of differentiating Eq. (5) with respect to ξ^3. This yields

$$\Delta\psi + \gamma_{23}\psi,_{23} - \gamma_{23},_3\psi,_2 + 2\gamma_{22},_3\psi,_3 + \gamma_{22},_{33}\psi$$
$$+ \gamma_{23}\phi,_{33} + 2\gamma_{23},_3\phi,_3 - \gamma_{33},_3\phi,_2 + \gamma_{23},_{33}\phi$$
$$= -\gamma_{33}(Ju),_2 + (JA),_3 \qquad (7)$$

Equations (2), (3), (6), and (7) are the boundary-layer equations for the radiused corner. The boundary conditions can be derived in a way similar to that used by Barclay and Ridha[4] for sharp corners and are found to be

$$\xi^2 = 0 \qquad u = \phi = \psi = 0 \qquad A = -\phi'/\cos^2\lambda$$

$$\xi^2 \to \infty \qquad u \sim 1 \qquad \phi \sim -1/2 I_2 u$$
$$\psi \sim 1/2[(\beta - I_1 u)\cos\lambda + I_2 u \sin\lambda] \quad A \sim 0$$

$$\xi^3 = 0 \qquad u,_3 = \psi,_3 = \phi = A = 0$$

$$\xi^3 \to \infty \qquad u = f' \qquad \psi = -1/2\cos\lambda^* f$$

$$\phi = -1/2\Big(\cos\lambda^*\xi^3 f' + I_3 f'$$
$$+ \sin\lambda^*\cos\lambda^*[\xi^2 f' - f]$$
$$- 1/2\sin\lambda^*\cos^3\lambda^*\beta f''\int_0^{\xi^2}\frac{t-\beta}{f''}\,dt\Big)$$

$$A = -\phi'/\cos^2\lambda^* \qquad (8)$$

where a prime denotes differentiation with respect to ξ^2, and $f(\xi^2)$ is the solution of $2f''' + \cos^2\lambda^* ff'' = 0$, satisfying $f(0) = f'(0) = 0$ and $f'(\infty) = 1$.

$$I_3 \equiv \int_0^\infty (\cos\lambda - \cos\lambda^*)\,d\xi^3$$

and

$$\beta = \lim_{\xi^2 \to \infty}(\xi^2 f' - f)$$

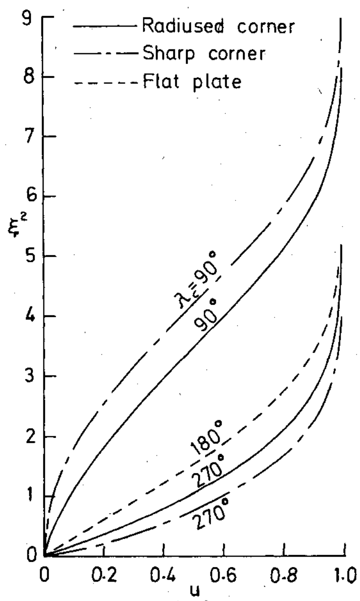

Fig. 2 Streamwise velocity component u in the symmetry plane of the corner.

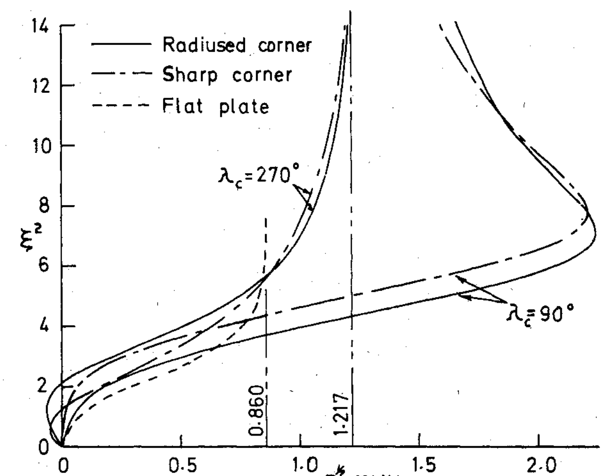

Fig. 3 Cross-flow velocity component $v(2)$ in the symmetry plane of the corner. All curves for $\lambda_c = 90$ deg and 270 deg asymptote to 1.217 as $\xi^2 \to \infty$.

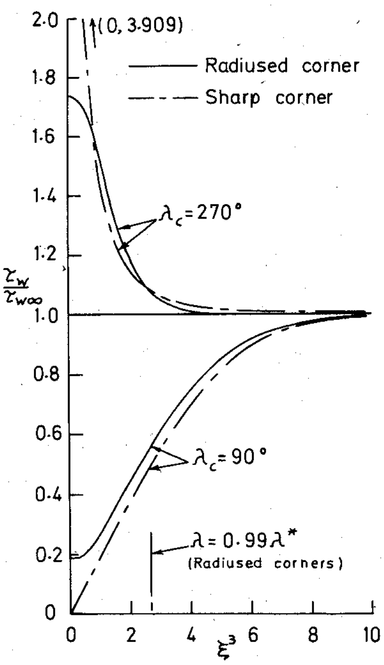

Fig. 4 Wall shear stress.

Equations (8) are slightly different in appearance from the corresponding results in Ref. 4 for the sharp corner, but it is easy to show that they represent exactly the same physical situation. In other words, the physical boundary conditions are dependent only on the angle λ^* and not on whether the corner is sharp or radiused.

Note that the equations determining the flow in a sharp streamwise corner are obtained directly from the above analysis by setting $\lambda=\lambda^*$ for all ξ^3 and replacing the conditions in Eq. (8) for $\xi^3=0$ with the more general form

$$u_{,3}-\sin\lambda u_{,2}=0, \quad \psi_{,3}-\sin\lambda\psi_{,2}+\sin\lambda\phi_{,3}=0$$

$$\phi=0, \qquad A=0$$

Choosing $\lambda=\lambda^* \tanh\xi^3$, the methods of Ref. 4 were used to solve Eqs. (2), (3), (6), (7), and (8) numerically for corners of included angles 90, 135, 225, and 270 deg (i.e., $\lambda^*=45$, 22.5, -22.5, and -45 deg).

The most significant results are the streamwise and Cartesian cross-flow velocity components u and $v(2)$ at the symmetry plane $y^3=\xi^3=0$ and the shear stress $\tau_w(\xi^3)$ on the wall $\xi^2=0$, where

$$\tau_w=\tau_{w\infty}\frac{\cos\lambda^*}{\cos\lambda}\frac{(\partial u/\partial\xi^2)_{\xi^2=0}}{f''(0)}$$

and

$$\tau_{w\infty}=\tau_w(\infty)=f''(0)/\cos\lambda^*$$

These quantities are shown in Figs. 2-4 for 90- and 270-deg corners. Included for comparison are the corresponding sharp corner results also obtained from the present program. These agree exactly with the results of Ref. 4 and therefore, as demonstrated in that paper, they are also in very good agreement with the 90-deg sharp corner results given in Ref. 3.

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Simple Relations for the Stability of Heated-Water Laminar Boundary Layers

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I. Introduction

RECENT theoretical and experimental studies suggest that surface heating in water holds promise for enhancing boundary-layer stability by retarding the growth of Tollmien-Schlichting instabilities, or by increasing the surface area over which two-dimensional, infinitesimal disturbances are damped. In the absence of a comprehensive theory of boundary-layer transition, linear stability theory currently provides the sole analytic guide for manipulating mean flow velocity profiles to delay transition.

Even for preliminary engineering applications, no single parameter describes the stability of a particular velocity profile. However, the minimum critical Reynolds number can be extremely useful, both as a precise measure of the extent over which all two-dimensional, infinitesimal disturbances are damped and as a simple qualitative surrogate for the stability characteristics of a particular velocity profile.[1]

Lin[2] developed a simple set of asymptotic relations for determining the minimum critical Reynolds number from the velocity profile of a constant property, laminar boundary layer. The Lin relations were widely used[3] until the advent of computer-based schemes for the solution of the Orr-Sommerfeld equation. In 1946, Lees and Lin[4] extended the original Lin relations to the compressible flow of air, and later, Dunn and Lin[5] and Mack[6] presented further compressible extensions of the original Lin analysis for the neutral curve. However, special difficulties associated with compressibility diminished the role of this analytic approach in high-speed gas dynamics, and numerical computation preempted analytic solution for the airflows.

For heated-water boundary layers, in practice, the situation is simpler than for compressible flows. Water density is nearly constant, temperature and viscosity fluctuations have little effect on stability, and the primary departure from the constant property incompressible flow originally considered by Lin is the variation of mean flow viscosity with temperature.[7,8]

In this Note, we show how the Lin relations and Dunn-Lin theory may be used, after slight modification, to estimate the minimum critical Reynolds number for heated boundary layers. The results of our modified Dunn-Lin analysis are then compared with values obtained from numerical solutions of the Orr-Sommerfeld equation for heated-water boundary layers. Once the accuracy of these new relations is established, they are then used to indicate the influence of changing temperature levels on the minimum critical Reynolds number.

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